



Article

Applicability of Geometric Brownian Motion and Geometric Fractional Brownian Motion to Forecast Share Prices of the Telecommunication Services Sector in Sri Lanka

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Abstract: The Brownian motion is a Mathematical concept that European botanist Robert Brown introduced in 1827 to study the behaviour of molecules. The Brownian motion concept was transformed into many versions, and Geometric Brownian Motion (GBM) and Geometric Fractional Brownian Motion (GFBM) are the latest transformations of this concept. The GBM and GFBM are mathematical models used to forecast the prices of stocks, commodities, etc. In this study, the GBM and GFBM were tested to estimate the share prices of telecommunication industry companies in Sri Lanka. The two sample companies were selected to represent 18% of the population of the telecommunication industry group. The five-year share prices were collected from sample companies: Sri Lanka Telecom PLC and Dialog Axiata PLC. The two models were implemented by estimating parameters such as the drift, the volatility, probability measurement and the time interval. In addition, the Hurst component was generated by a MATLAB program for GFBM. This study concluded that GBM is the most accurate model for forecasting share prices of the telecommunication industry group with minimum mean absolute percentage error (MAPE).

Keywords: GBM, GFBM, MAPE.

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1. INTRODUCTION

1.1 The Stock Exchange

The stock market is a vital contributor to the economy of any country. The stock market or stock exchange is a market used for transacting shares. The transactions are issuing shares

by public companies and buying and selling the shares. These financial activities are performed through stock exchanges (whether physical or electronic) or over-the-counter (OTC) marketplaces according to the set of legislation. Since the evolution of technology, all financial activities are now conducted via online platforms.

The operations in the stock exchange can be divided into two groups: primary and secondary. In the primary market, the stock market works with organisations to issue and offer their shares to the general public for the first time through the process of an initial public offering (IPO). In the secondary market, these primary shares are transacted through a large number of investors according to the standard rule of “supply and demand”. As an advantage of the stock market, the public companies can raise their capital while the public grabs the opportunity for ownership of a public company. In addition, dividend payments are made to shareholders.

There may be one or more stock exchanges in a country. The stock exchange of Sri Lanka is named “Colombo Stock Exchange (CSE)”. The CSE was not built overnight, and it dates back to 1986. Its inception goes back to 1896, when it was known as the “Colombo Share Brokers Association (CSBA)”, and since then it has passed through many milestones. There are 297 public companies registered under the Companies Act No.07 of 2007. These public companies represent 20 Global Industry Classification Standard (GICS) industry groups with a market capitalisation of Rs. 5815.46 billion as at the 31st of January 2022. The public companies integrated under the Companies Act No.7 of 2007 or any other state corporations, inaugurated under the laws of Sri Lanka or initiated under the laws of any other state (subject to Exchange Control approval), are eligible for the registration of CSE according to the Securities & Exchange Commission Act No.36 of 1987 and the legislation of the exchange. The CSE maintains two indexes: the All Share Price Index (ASPI) and S&P SL20, and manages two main systems: central depository system and an automated trading system. The fifteen broker companies have been engaged with CSE by empowering the vision “To be the preferred choice for the creation of wealth and value”.

This study focuses on the two companies in the telecommunication services industry group: Sri Lanka Telecom PLC and Dialog Axiata PLC. According to the GICS hierarchy, telecommunication services are the leading industry group in the communication services sector. This industry group is divided into two industries: diversified telecommunication services and wireless telecommunication services. The diversified telecommunication services industry has two sub-industries: alternative carriers and integrated telecommunication services. The wireless telecommunication services industry has one subindustry. The sample industry group of this study is presented in Figure 1.

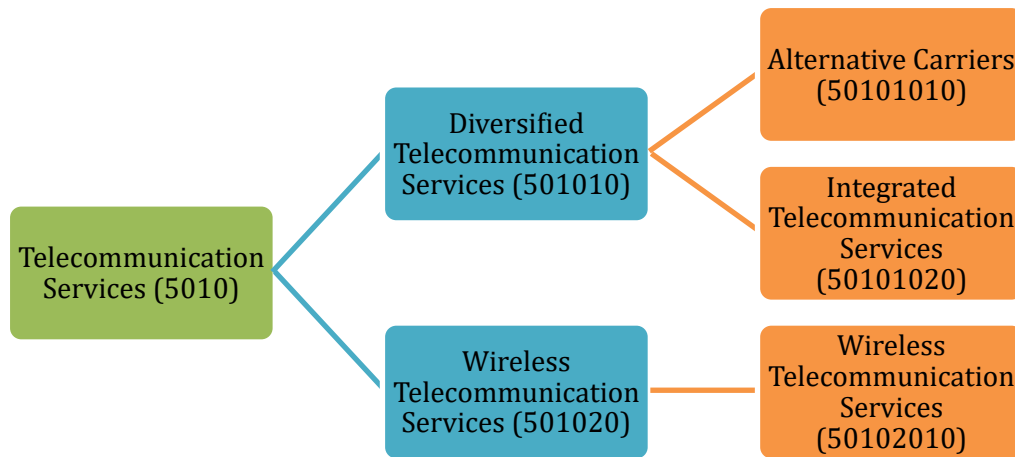


Figure 1: The industry group of telecommunication services

Source: Author's creation, 2024

1.2 Sri Lanka Telecom PLC

Sri Lanka Telecom PLC is a leading information & communication technology service provider in Sri Lanka. SLT serves a consumer base of over nine million islands-wide by empowering its latest technology with ultimate-speed fibre, copper and wireless access networks. SLT has vastly contributed to the Sri Lankan economy, as well, by diversifying its strategies of land and portable telephone, broadband, data services, Internet Protocol Television (IPTV), cloud computing and hosting services, and networking solutions. SLT has subsidiaries such as eChanneling PLC, Mobitel (Pvt) Ltd and the Sri Lanka Technological Campus. SLT has a market capitalisation of 1.69% in CSE by issuing 1,804,860,000 shares.

1.3 Dialog Axiata PLC

Dialog Axiata PLC is a subsidiary of Axiata Group Berhad (Axiata) and a well-known mobile telecommunication service provider in Sri Lanka. Dialog serves through its high-level portable communication and high-speed mobile broadband services to a consumer base of over 17 million Sri Lankans. Dialog also demonstrated the first 5G pilot transmission in South Asia. Dialog has a market capitalisation of 1.99% in CSE by issuing 117,399,866 shares.

1.4 Brownian Motion Concept

The Brownian motion is the concept of this study. It was founded by a European botanist, Robert Brown, in 1827. He introduced this concept to study the behaviour of molecules based on the kinetic-molecular conception of matter. In 1905, Albert Einstein developed a scientific approach, and Norbert Wiener formulated a mathematical expression for it in 1918. The French Mathematician Bachelier improved this concept for modelling share price fluctuations. The Geometric Brownian Motion (GBM) model is the latest outcome of this

concept, and it can be generalised by the Geometric Fractional Brownian Motion (GFBM) model.

This study refers to the applicability of the Brownian motion concept to the telecommunication services sector in Sri Lanka. This Brownian motion concept is commonly used in modelling stock price movements due to its stochastic nature. Therefore, It has broadly tested the GBM and GFBM in order to select the most accurate model for forecasting share prices of the telecommunication services sector.

1.5 Research Objectives

Primary Objective

To test the most accurate model between GBM and GFBM for forecasting share prices in the telecommunication service sector companies in Sri Lanka.

Secondary Objectives

- To study the concept of Brownian motion in the field of Financial Mathematics.
- To study the GBM and GFBM models.
- To estimate the parameters of sample companies for GBM and GFBM models.
- To apply both models to sample companies.
- To analyse the most accurate model by comparing the Mean Absolute Percentage Error (MAPE).

2. LITERATURE REVIEW

Ibrahim et al. (2020) investigated the accuracy of GBM and GFBM in modelling Malaysia's crude palm oil price simulation. In their research, persistent and anti-persistent behaviour across different periods was displayed. The Hurst component was calculated in three different methods: absolute moment, aggregate variance, and the Higuchi method.

According to Areerak (2014), the Brownian Motion model with adaptive parameters (BMAP) and the Fractional Brownian Motion model with adaptive parameters (FBMAP) were tested to forecast future share prices in two companies in the Thai stock market. The ADVANC Info Service Public Company Limited (ADVANC) and Land and Houses Public Company Limited (LH) closed prices are discussed in the paper. He concluded that FBMAP was the most suitable model for forecasting the share prices of his sample companies. As a new point, the parameters were adapted to avoid the effect of closed days of the share market.

The review looks at the way South African monetary business sectors behave with respect to the GBM process. It utilizes the daily, weekly, and monthly stock returns time series of a few significant protections exchanging the South African monetary market, all the more explicitly the US dollar/Euro, JSE ALSI Total Returns Index, South African All Bond Index,

Anglo-American Corporation, Standard Bank, Sasol, US dollar Gold Price, Brent spot oil cost, and South African white maize not so distant future. It likewise uncovers the presence of both long memory and random walk or GBM in the South African monetary business sectors returns when the Hurst component estimation is utilised and observes that the Currency market is the most efficient of the South African monetary business sectors. That's what the review infers, albeit a few presumptions that the cycle is abused, the Brownian motion as a model in South African monetary business sectors can't be dismissed. It very well may be acknowledged in certain occasions if a few boundaries, for example, the Hurst component, are added (Karangwa, 2008).

According to Agustini et al. (2018), the GBM model is the most accurate prediction technique for forecasting share prices, with a 95% confidence level, as evidenced by an MAPE of less than 20%. In this study, the share prices are obtained from the Indonesian stock exchange under the Jakarta Composite Index, which includes Charoen Pokphand Indonesia Tbk, Harum Energy Tbk, Media Nusantara Citra Tbk, PP London Sumatra Indonesia Tbk, Vale Indonesia Tbk, Indo Tambangraya Megah Tbk, and Indocement Tunggul Prakasa Tbk.

It is realised that the market records of Saudi Arabia, which is named the Tadawul All Share Index (TASI), mirror the exhibition of economic development and monetary stability of Saudi Arabia. Consequently, determining the performance is very significant. In this experimental review, we determined the daily index prices of TASI for the year 2018. To act on this, we relied upon two models, including GBM and GFBM. Further, the estimation of each model was acquired dependent on three unique approaches of calculating volatility, such as simple volatility, log volatility and stochastic volatility. In the interim, the assessment of the presentation of each model was determined by utilizing mean absolute percentage error (MAPE). The outcomes uncovered that all models have high precision with paltry distinction. This demonstrates that every model can be accustomed to anticipating the exhibition of TASI (Alhagyan & Alduais, 2020).

Si and Bishi (2020) used GBM to forecast share prices in the Bombay Stock Exchange (BSE). He proved it with a MAPE value of less than 10% i.e. 5.41%. Before forecasting the stock price using the GBM model, the Kolmogorov-Smirnov test and the Q-Q plot technique were run on the sample data to verify that the data are normally distributed and practicable to forecast.

Until the 1970s, most Muslim people were not associated with the share market venture due to the Islamic prohibitions of specific activities. During the 1990s, a significant advancement happened in a portion of the arrangements of regulation related to religious equity ventures, and from that point forward, the Islamic equity funds began working. The securities exchange is where securities of organisations are traded under strict standards, guidelines and rules. The Malaysian stock exchange is called Bursa Malaysia. Syria-based equity is essentially portions of the organisation meeting the standards of Islamic law. Roughly 88% of the ongoing effects of posting on BM are Syria agreeable and address two-thirds of the market capitalisation of Malaysia. The securities exchange follows Brownian

motion, where the assets are ceaselessly assigned in tiny time spans and positions, and the value of the assets of the country varies by an irregular number. The paper conjectures that Syria's counter is in four areas: development, money, agriculture and trading services. The technique involved was the Geometric Brownian motion as far as instability models. This strategy can be applied by the analyst to gauge the stock prices (Omar & Jaffar, 2011).

This empirical literature was helpful in this research study to develop the theoretical background. Moreover, the structure of this study is sketched by using research that was conducted in many different countries, not only for the stock market, but for the commodity market.

According to the literature, these two approaches (GBM & GFBM) were used in Financial Mathematics in forecasting share prices in several sectors in many countries. Therefore, the determination of the most accurate model among GBM and GFBM for forecasting share prices of telecommunication services sector companies in Sri Lankan context could be identified as the empirical research gap of this study.

3. RESEARCH METHODOLOGY

3.1 Population

The population of the research is based on the Global Industrial Classification Standard (GICS). GICS was developed by an independent international financial data and investment services company called S&P Dow Jones Indices. The GICS hierarchy consists of 11 sectors, 24 industry groups, 69 industries and 158 sub-industries. Therefore, GICS has become the universal classification for financial investment research, industry analysis, portfolio management and asset allocation. According to the methodology of GICS, a single sector can be categorised as shown in Figure 2.



Figure 2: The structure of GICS
Source: Author's creation, 2024

There are two industry groups in the communication services sector, and one of these industry groups is based on the population. The hierarchy of the communication services sector can be shown in Figure 3.

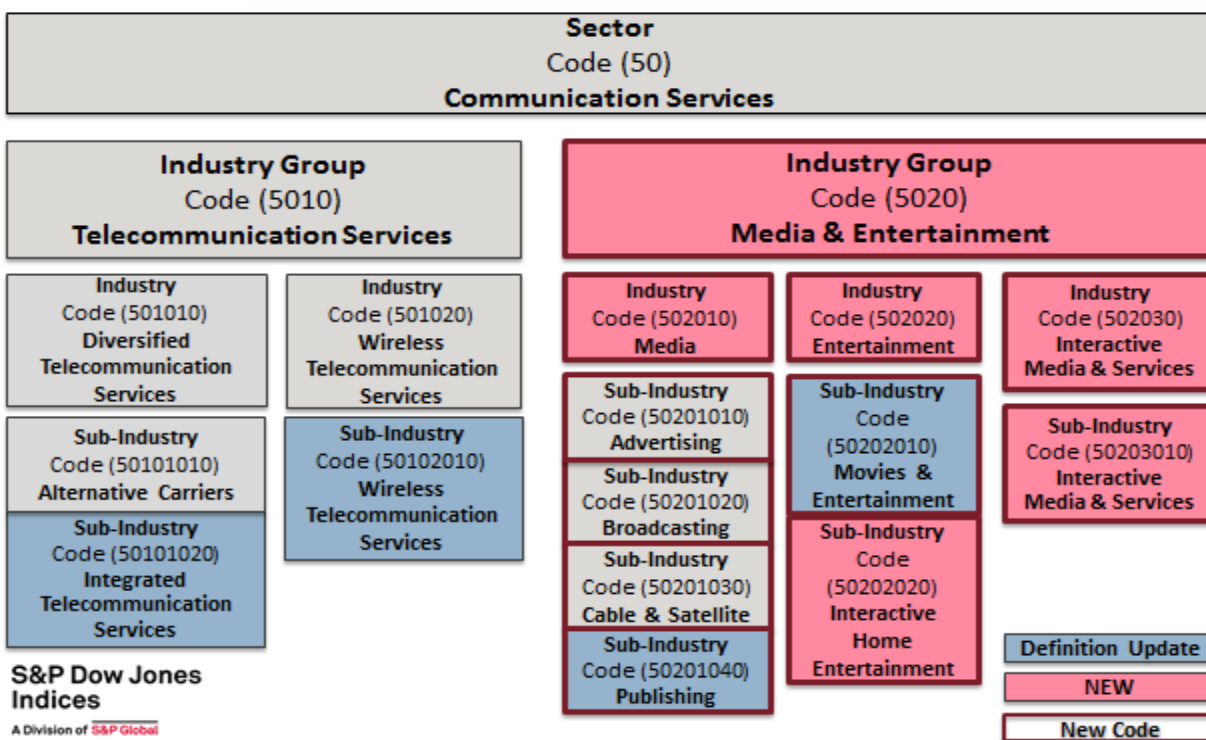


Figure 3: The hierarchy of communication services

Source:

The industry group “Telecommunication Services” consists of two industries: diversified telecommunication services and wireless telecommunication services. The industry of diversified telecommunication services has two sub-industries called alternative careers and integrated telecommunication services. The 11 companies which have obtained the license of the Telecommunication Regulatory Commission of Sri Lanka (TRC) were selected as the population of the research. These 11 companies are as follows.

- Sri Lanka Telecom PLC
- Lanka Bell Ltd
- Dialog Broadband Networks (Pvt) Ltd
- Hutchinson Telecommunication Lanka (Pvt) Ltd
- Bharti Airtel Lanka (Pvt) Ltd
- Mobitel (Pvt) Ltd
- Dialog Axiata PLC
- TATA Communications Lanka Ltd

- Lanka Communication Services (Pvt) Ltd
- Societe International Telecommunications Aeronautiques (SITA)
- Lanka Education & Research Network (LEARN)

3.2 Sample

The two sample companies are selected from Eleven telecommunication companies serve the population. These two companies are the only two telecommunication companies that issues shared with the general public. The sample proportion can be calculated as 18.2% $\left(\frac{2}{11} \times 100\%\right)$. The following two companies, which are registered in the Colombo Stock Exchange (CSE) were selected as the sample.

- Sri Lanka Telecom PLC
- Dialog Axiata PLC

3.3 Sampling Method

In statistics, there are four non-probability sampling methods, such as convenience sampling, judgmental /purposive sampling, snowball sampling and quota sampling. The purposive sampling is a sampling method that is purely built according to the purpose of the research. The accuracy of a mathematical model for forecasting share prices is tested in this research. Therefore, public limited companies should be selected for the sample. The two companies which are registered in CSE are purposively selected among the population of 11 telecommunication services companies.

3.4 Data Collection Method

The two kinds of data are introduced in the research methodology. They are primary data and secondary data. Primary data is obtained directly by the data source, and secondary data is the data that has been gathered in the past by someone else and shared publicly. The share prices of sample companies are taken from [the www.investing.com](http://www.investing.com) website. Therefore, the data of the research is secondary and collected in an automated way by entering the time duration.

The daily share prices of each company for the five-year period (from 01st Jan 2016 to 31st Dec 2020) is collected. The actual data for SLT is from 05/01/2016 to 31/12/2020, and the actual data for Dialog is from 04/01/2016 to 31/12/2020. The details of share prices can be shown in Table 1.

Table 1: No of share prices per year

Company	2016	2017	2018	2019	2020	Average
---------	------	------	------	------	------	---------

SLT	235	233	237	237	208	230
Dialog	239	241	240	241	209	234

Source: Author's creation, 2024

3.5 Tools

The MATLAB software package is used to calculate the Hurst component, and MINITAB is used for the descriptive analysis. The entire model is analysed with Microsoft Excel.

4. ANALYTICAL FRAMEWORK

The theoretical formulas that were used for the analysis are clearly presented under the following sub-topics.

4.1 GBM Model

The final formula for GBM can be presented as follows. The $S(t + \Delta t)$ be the forecasted share price.

$$S(t + \Delta t) = S(t) \cdot \exp \left[\left(\mu - \frac{1}{2} \sigma^2 \right) \Delta t + \sigma z \sqrt{\Delta t} \right] \quad (1)$$

Where

- μ - Annual Drift
- σ - Annual Volatility
- z - Inverse of the Standard Normal Probability
- Δt - Time difference between two consecutive trading periods (years)
- $S(t)$ - Share price in the previous time period

In order to forecast share prices using GBM, the parameters of annual volatility, annual drift, and the standard normal probability should be estimated.

Calculation of Annual Volatility

The volatility is a fixed parameter of a stock and is expressed annually. The volatility gives an opinion on the stability of share prices. The high volatility refers to the stock price, which changes continuously within a relatively high interval. The most common and accepted method of measuring the stock volatility is the standard deviation of the price returns. The following algorithm is used to calculate the annual volatility.

Let S_i be the stock price at the end i^{th} merchandising period and u_i be the logarithm of the daily return as follows.

$$u_i = \ln \ln \left(\frac{S_i}{S_{i-1}} \right) \quad \text{for } i = 1 \dots n \quad (2)$$

The $\underline{u}$ be the mean and v be the standard deviation of u_i . The annual volatility (σ) can be obtained by using the estimated $\underline{u}$ and v .

$$\text{annual volatility } (\sigma) = \frac{v}{\sqrt{\Delta t}} \quad (3)$$

Calculation of Annual Drift

$$\text{Annual Drift}(\mu) = \frac{\underline{u}}{\Delta t} + \frac{\sigma^2}{2} \quad (4)$$

Calculation of Probability Measurement

A continuous random variable S_t act in accordance with a lognormal distribution, if its natural logarithm, $\ln \ln (S(t))$ undergoes a normal distribution. Here, $S(t)$ follows a lognormal distribution, since $(S(t))$ follows the normal distribution. Let the mean and the standard deviation of $(S(t))$ The data set is m & s

$$\ln \ln (S(t)) \sim N(m, s^2) \Rightarrow S(t) \sim \text{lognormal}$$

Probability density function of log-normal distribution,

$$f(S(t), m, s) = \frac{1}{\sqrt{2\pi} \cdot s \cdot S(t)} \exp \left[-\frac{[\ln \ln (S(t)) - m]^2}{2s^2} \right] \quad (5)$$

Where,

Mean	$\mu' = \exp \left[\left(m + \frac{s^2}{2} \right) \right]$
Variance	$\sigma'^2 = [e^{s^2} - 1] \exp[(2m + s^2)]$

The standard normal probability (z) for each share price at t can be obtained by the inverse of the probability density function, which is calculated in formula 5.

4.2 GFBM Model

The final formula for GFBM can be presented as follows. The $S_H(t + \Delta t)$ be the forecasted share price.

$$S_H(t + \Delta t) = S(t) \cdot \exp \left[\left(\hat{\mu} \Delta t - \frac{1}{2} \hat{\sigma}^2 (\Delta t)^{2H} + \hat{\sigma} z \sqrt{(\Delta t)^{2H}} \right) \right] \quad (6)$$

Where,

- H - Hurst component
- $\hat{\mu}$ - Annual Drift with Hurst component
- $\hat{\sigma}$ - Annual Volatility with Hurst Component

Δt - Time difference between two consecutive trading periods (years)

$S(t)$ - Share price during the previous time period

In order to forecast share prices using GFBM, the parameters of $H, \hat{\mu}, \hat{\sigma}$ should be estimated. The calculation of z is similar to the GBM model.

Calculation of Hurst Component

There are nine methods to estimate the Hurst component. The rescaled range analysis is the most used and most accurate method to estimate the Hurst component. In this study, the rescaled range analysis is applied to find the Hurst component and the algorithm is given in Figure 4.

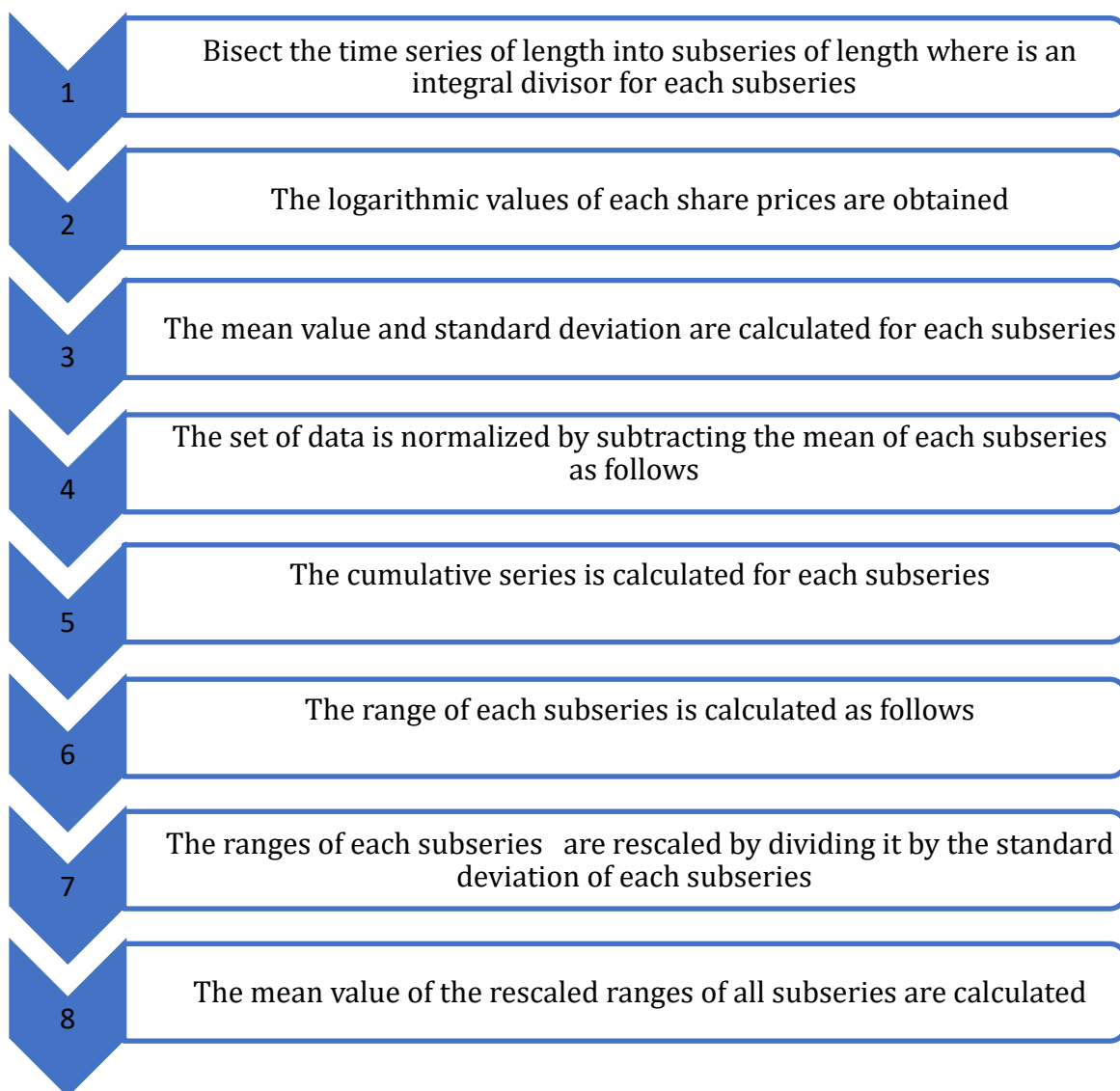


Figure 4: Algorithm of Hurst Component

Source: Author's creation, 2024

The above algorithm is processed by the MATLAB program. The $(R/S)_l$ components for relevant divisions of time series can be obtained by the program.

The $(R/S)_l$ statistic follows the relation, $(R/S)_l = c * l^H$. Thus, the value of H can be generated by using a simple linear regression. The variable $\log \log (l)$ as the independent, and the variable $\log \log ((R/S)_l)$ is the dependent. The slope of the resulting Equation 7 is the value of the Hurst component.

$$\begin{aligned} \left(\frac{R}{S}\right)_l &= c * l^H \\ \ln \ln \left[\left(\frac{R}{S}\right)_l\right] &= \ln \ln [c * l^H] \\ \ln \ln \left[\left(\frac{R}{S}\right)_l\right] &= \ln \ln [c] + H * \ln \ln [l] \\ \ln \ln \left[\left(\frac{R}{S}\right)_l\right] &= H * \ln \ln [l] + \ln \ln [c] \end{aligned} \quad (7)$$

Calculation of Volatility with the Hurst Component

The volatility formula is slightly varied due to the Hurst component as follows.

$$\hat{\sigma} = \frac{v}{\sqrt{(\Delta t)^{2H}}} \quad (8)$$

Calculation of Drift with the Hurst Component

The drift with the Hurst component can be obtained by substituting values for formula 9.

$$\hat{\mu} = \frac{u}{\Delta t} + \frac{\hat{\sigma}^2}{2} \quad (9)$$

4.3 Calculation of MAPE

This is the most important calculation for the final result. The minimum MAPE values are obtained to choose the most accurate model. The MAPE is calculated by using the following formula.

$$MAPE = \frac{1}{n} \sum_{i=1}^n \frac{(A-F)}{A} \quad (10)$$

Where,

A	-	Actual share price
F	-	Forecasted share price
n	-	Number of data points

The interpretation of MAPE is shown in Table 2.

Table 2: Interpretation of MAPE

MAPE Value	Interpretation
< 10%	A highly accurate forecasting
10 – 20%	A good forecasting
20 – 50%	A reasonable forecast
> 50%	An inaccurate forecast

Source: Brătian et al, 2021

5. ANALYSIS

5.1 Descriptive Analysis

The abstract of the descriptive analysis is presented in Table 3. The number of data points of SLT and Dialog is 1150 and 1170, respectively. The mean share price and standard deviation of SLT are 30.232 and 5.65, respectively. The mean share price and standard deviation of Dialog are 11.514 and 1.375, respectively.

Table 3: The summary of descriptive analysis

Statistic	SLT	Dialog
No of variables	1150	1170
Mean	30.232	11.514
Standard Deviation	5.650	1.375
Variance	31.927	1.892
Minimum Value	19.000	8.000
First Quartile	26.100	10.600
Median	30.000	11.400
Third Quartile	34.125	12.400
Maximum Value	46.400	14.800
Mode	30	11.4
Frequency for the mode	39	59
Skewness	0.14	0.29
Kurtosis	-0.64	-0.40

Source: Author's creation, 2024

5.2 GBM & GFBM Model

The estimated parameters for GBM and GFBM models are shown in Table 4. The estimation of Δt is taken by assuming the average number of days per year for SLT and Dialog as 230 and 234, respectively.

Table 4: Parameter estimation for GBM & GFBM

Parameter	SLT	Dialog
$\underline{u}$	-0.000273	0.000134
$\underline{v}$	0.024671	0.014378
σ	0.374	0.2199
μ	0.007148	0.0555
Δt	$\frac{1}{230}$	$\frac{1}{234}$
H	1.034817	1.003783
$\hat{\sigma}$	6.857155	3.4250
$\hat{\mu}$	23.4475	5.8967

Source: Author's creation, 2024

After applying time-independent variables to GBM, the final formulas for SLT and Dialog are presented in 11 and 12, respectively.

$$S(t + \Delta t) = S(t) \cdot \exp[-0.000273 + 0.0247z] \quad (11)$$

$$S(t + \Delta t) = S(t) \cdot \exp[0.000134 + 0.0144z] \quad (12)$$

After applying time-independent variables to GFBM, the final formulas for SLT and Dialog are presented in 13 and 14, respectively.

$$S_H(t + \Delta t) = S(t) \cdot \exp[0.1016 + 0.0247z] \quad (13)$$

$$S_H(t + \Delta t) = S(t) \cdot \exp[0.0251 + 0.0144z] \quad (14)$$

The time series plots of actual and forecasted share prices using GBM and GFBM of SLT are demonstrated in Figure 5 and Figure 6, respectively. Furthermore, the time series plots of actual and forecasted share prices using GBM and GFBM of Dialog are demonstrated in Figure 7 and Figure 8, respectively.

5.3 Graphical Representation of Forecasted Data

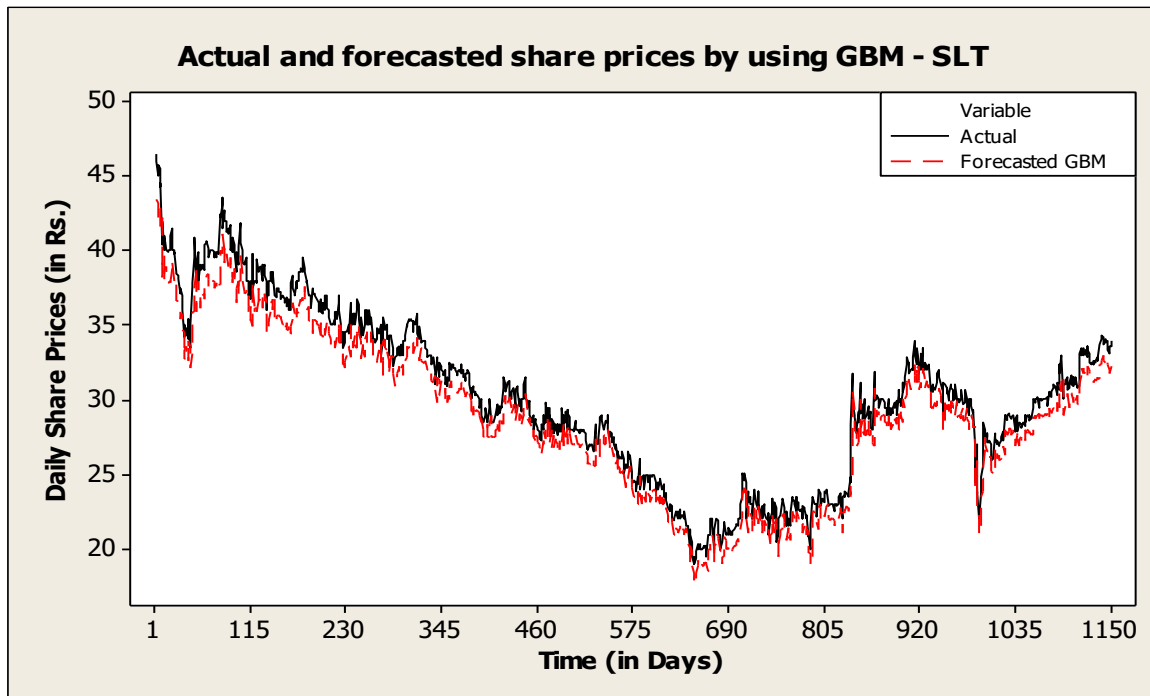


Figure 5: Actual and forecasted share prices using GBM – SLT
Source: Author's creation, 2024

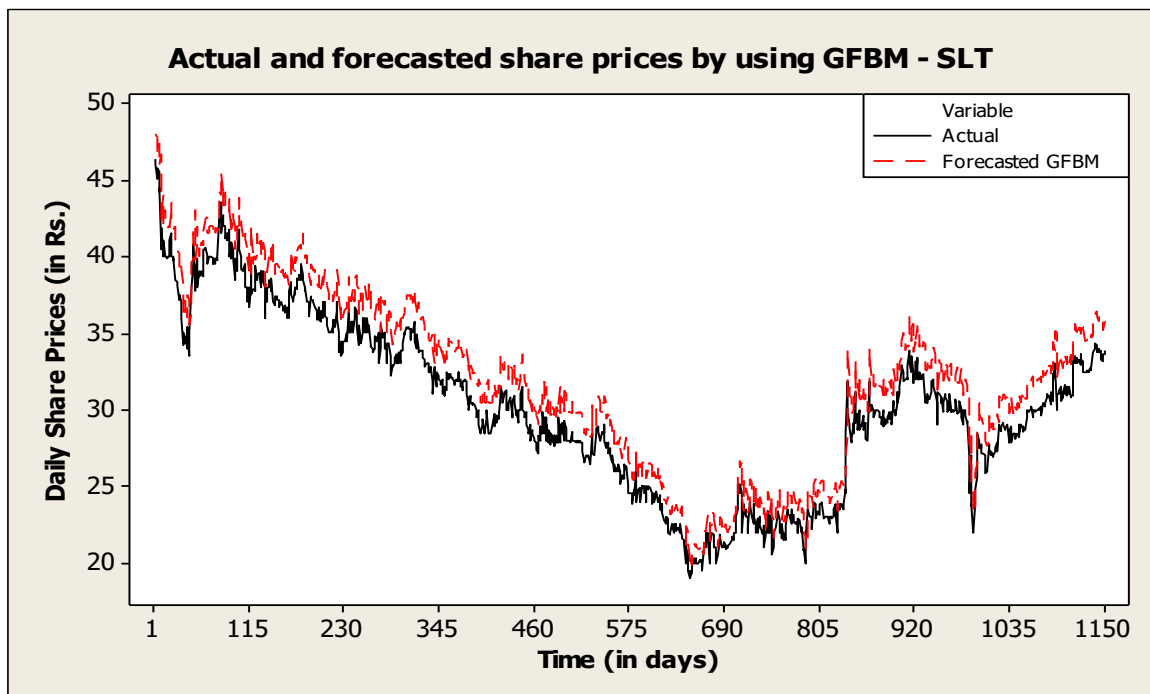


Figure 6: Actual and forecasted share prices using GFBM – SLT
Source: Author's creation, 2024

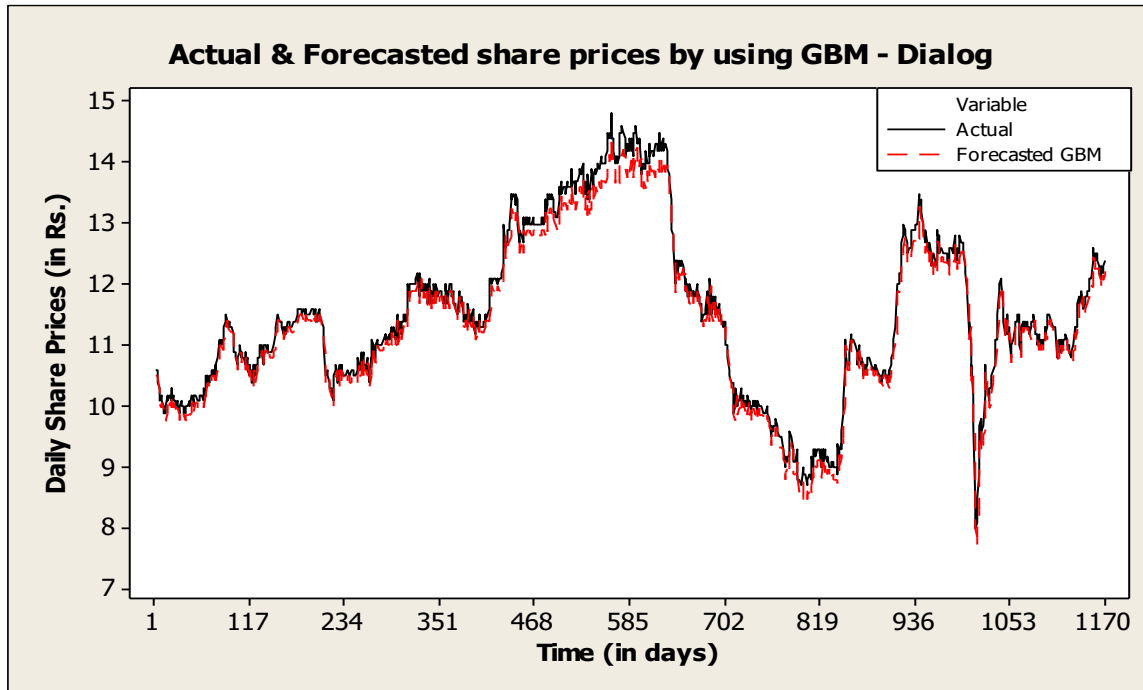


Figure 7: Actual and forecasted share prices using GBM – Dialog
 Source: Author's creation, 2024

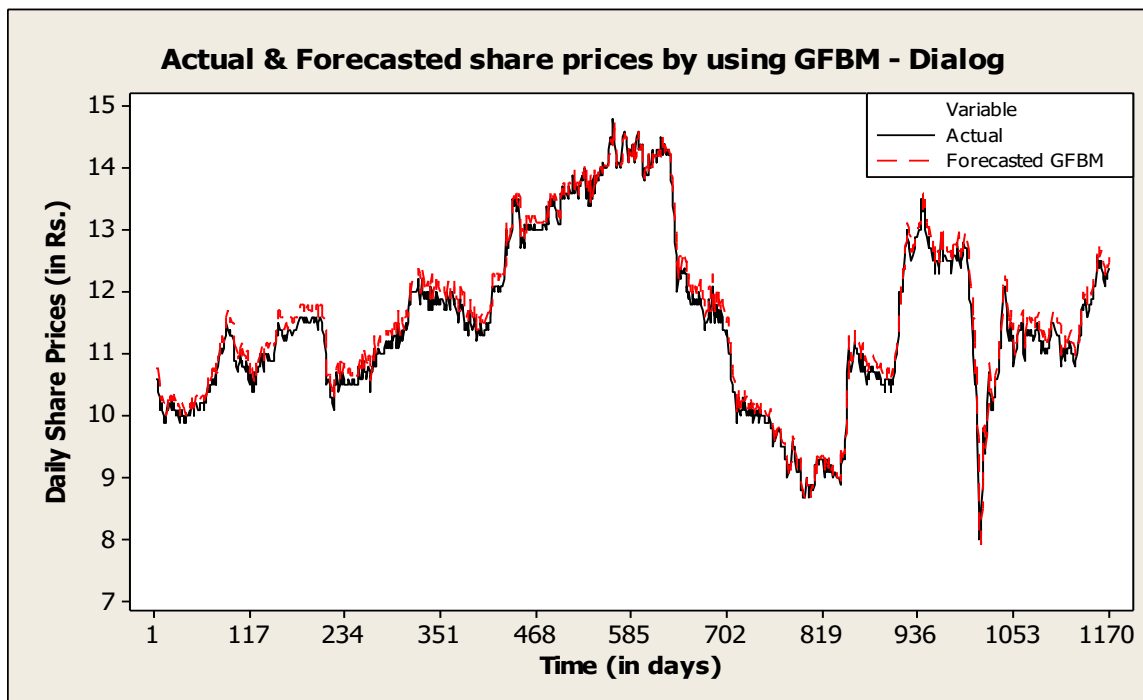


Figure 8: Actual and forecasted share prices using GFBM – Dialog
 Source: Author's creation, 2024

6. DISCUSSION AND CONCLUSION

In this study, two mathematical models are tested to forecast share prices. The daily share prices for the 5 years of the two companies in the telecommunication sector was analysed. In this chapter, the summary of the research is presented.

As the first sample company, 1150 daily share prices for five years are obtained from Sri Lanka Telecom PLC. SLT has 230 trading days per year. The mean share price is Rs 30.232, and the variance is 31.927. The share prices varied from Rs 19 to Rs 46.4, representing the minimum and maximum values, respectively, within the five-year time period.

As the second sample company, 1170 daily share prices for five years are obtained from Dialog Axiata PLC. Dialog has 234 trading days per year. The mean share price and variance become Rs 11.514 and 1.892, respectively. The minimum and maximum share prices are Rs . 8 and Rs . 14.8.

By comparing these descriptive statistics of both companies, the mean value of a single share of SLT is greater than that of Dialog. Also, the high liquidity of SLT can be noticed more than that of Dialog. Both companies were affected by COVID-19 pandemic, and it led to a reduction of average trading days per year. Since the CSE was closed at the beginning of 2020, there were only 208 and 209 trading days in 2020 for SLT and Dialog, respectively.

As the ultimate result of this study, the share prices of each company are estimated using GBM and GFBM models. In order to estimate share prices by using GBM, the parameters of $S(t)$, μ , σ , Δt and z should be calculated for each company. The parameters of μ , σ and Δt are independent of time and have a single constant value for the whole time series. The parameters of $S(t)$ and z are dependent variables of time and have a series of values.

In order to estimate share prices by using GFBM, the parameters of $S(t)$, $\hat{\mu}$, $\hat{\sigma}$, Δt and z should be calculated. The parameters of $\hat{\mu}$, $\hat{\sigma}$ and Δt are independent of time and have a single constant value for the whole time series. The parameters of $S(t)$ and z are dependent variables of time and have a series of values.

The estimated share prices for both companies were obtained using GBM and GFBM. The mean absolute percentage error can be presented in Table 5.

Table 5: MAPE values

Company	GBM	GFBM
SLT	4.2%	6.3%
Dialog	1.5%	1.6%

Source: Author's creation, 2024

Since the minimum MAPE values, GBM is the most accurate model for forecasting the share prices of both companies, SLT and Dialog.

The GBM is the most accurate model between GBM and GFBM for forecasting share prices of the telecommunication sector in Sri Lanka.

6.1 Recommendations for Future Researches

The four stakeholders can be introduced through this study. They are:

- Two sample companies
- The other telecommunication companies in the population
- The current shareholders of the sample companies
- The people who wish to invest in the telecommunication sector

The recommendation which can be given to all the above stakeholders are to forecast share prices by using the GBM model and make rational decisions. Investors can use GBM's accurate forecasts to better assess the risk and volatility associated with telecommunication stocks, and it may lead to portfolio diversification and risk mitigation. Policymakers and regulatory bodies can use the volatility and drift estimates from GBM to monitor market stability and identify periods of excessive speculation or instability in the telecommunication sector. The telecommunications sector is critical for infrastructure and digital economy growth. Reliable stock price forecasts can attract more foreign direct investment, aiding in the economic advancement of Sri Lanka.

The GBM and GFBM models are tested to forecast the share prices of telecommunication sector companies in Sri Lanka. The same models can be used to forecast share prices of other sectors of the GICS classification in CSE.

In this research, the rescaled range analysis was used to estimate the Hurst component. There are more than eight methods to calculate the Hurst component, such as the aggregate variance method, the Higuchi method, and the detrended fluctuation analysis, etc. This research can be followed by estimating the Hurst component by using other methods.

The concept of GBM and GFBM can be applied to forecasting the prices of the commodity market. Although it is not practicable for Sri Lanka, it can be implemented for the commodity markets in other countries.

The research can be enhanced to test these GBM and GFBM models when the market is affected by external factors such as the COVID-19 pandemic, economic crisis and terrorist attacks.

The time series forecasting techniques, such as Auto Regressive Integrated Moving Average (ARIMA) models, can be used for forecasting share prices, as well as the Geometric Brownian motion concept. But GBM and GFBM are widely used in financial mathematics because they are designed to model the stochastic behaviour of asset prices, capturing the random fluctuations and volatility present in financial markets. GBM assumes a constant drift and volatility, aligning with the efficient market hypothesis, which suggests that price movements are largely random and unpredictable.

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CONFLICT OF INTEREST

The authors declare no conflicts of interest.

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